



## APPLICATION NOTE 31

### Expanded Nyquist Sampling theory

#### Introduction

With an increasing number of products that utilise digital sampling techniques to provide measurement functions previously achieved with analogue circuits; we inevitably see an increase in technical arguments debating the relative merit of one or other approach to digital electronics.

In this guide, we will explain a technique developed by N4L that overcomes a problem faced by most digital sampling measurement instruments and in making this process clear, we hope to dispel some commonly held misconceptions regarding digital sampling theory.

#### Sample rate versus sample accuracy

Before moving onto the subject of nyquist sampling, the issue of sample rate versus accuracy is worth a brief review. In the world of digital sampling, issues of sample rate, bit resolution, linearity, noise rejection, bandwidth and so on are used in conjunction with mathematical algorithms and sampling theories to argue the benefit of many different designs. The reality is that different approaches suit different applications and it is therefore wrong for anyone to argue that one aspect of a design is always more important than another. It is nonetheless useful to remain mindful of two important facts:

- A. **Within any cost band, A/D devices with higher sample rate always achieve lower resolution/accuracy.**

Digital Oscilloscopes are a good example of this, where the bit resolution and linearity of their high speed input cannot achieve the accuracy of instruments designed for lower frequency applications.

- B. **The accuracy of any wideband measurement instrument is primarily dictated by the analogue design and calibration.**

While the digital design of any digital instrument is clearly important, the accuracy of all these instruments is dictated by the analogue design and calibration. The sample rate or 'number of bits' are largely meaningless in defining instrument accuracy when the integrity of digital data is completely dependent upon the analogue attenuator or shunt from which measurements are being derived. It follows that in all measurement instruments, you are only as accurate as you can prove by calibration, not as accurate as you claim or can hope to derive using digital theory.

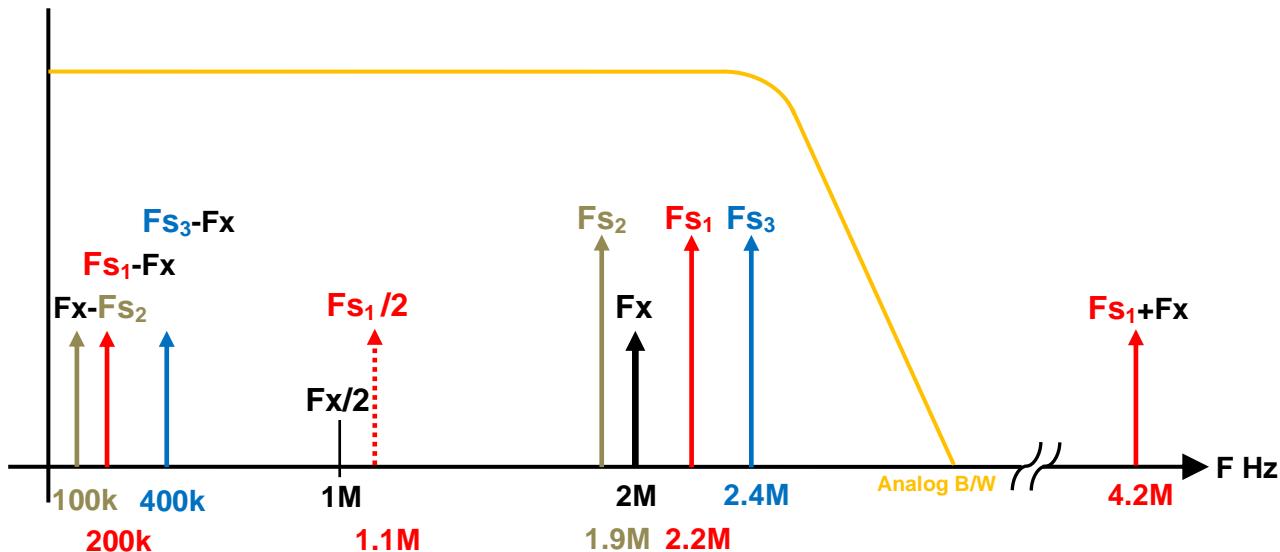
#### Nyquist–Shannon sampling theorem

Most engineers working in the field of AC electronics are familiar with the term 'Nyquist theory' or more accurately, 'Nyquist-Shannon sampling theorem'. The term is regularly introduced with respect to digital sampling instruments where people will confidently state that alias components are avoided by using a sample rate that is at least twice the frequency of interest. In fact, this is a common misinterpretation of the theorem which actually refers to the 'Bandwidth' not 'Frequency' of interest. So, provided that a frequency component is within a 'bandwidth' of half the sample rate, the frequency can be correctly measured.

It is based upon this true definition of the Nyquist-Shannon sampling theorem that N4L engineers developed a unique sampling technique we call Expanded Nyquist Sampling.

To explain the principal of our sampling technique, we follow with a diagram in which we show a frequency of interest ' $F_x$ ' at 2MHz which is the maximum specified frequency of the PPA45/55 series power analyzers. These analyzers have a 'nominal' sample rate of 2.2Ms/s and while this sample rate is higher than most comparable products, anyone who considers nyquist theory may ask how measurements can be made at this frequency without the risk of aliasing?

An alias component can be anticipated if we know both the frequency of interest and the sample rate being used to analyse the signal. It follows that an alias component can be avoided if a sample rate is used at which an alias will not occur. By design, all PPA series analysers from N4L have the ability to adjust A/D sample rate in real time, so, the challenge is to precisely determine the frequency of interest and then ensure that the selected sample rate will not alias on this frequency.



#### Principles:

1. We know from digital sampling theory that a frequency component of  $< F_x/2$  [in this example 1MHz] can be accurately derived with a sample rate  $F_x$  [in this example 2Ms/s]. It follows that any frequency components up to 1MHz will never alias.
2. From the detailed meaning of nyquist theory which refers to sampling bandwidth not sample rate as commonly believed, we know that frequency components within a frequency range  $F_x - (F_x/2)$  [1MHz] to  $F_x + (F_x/2)$  [3MHz] can be derived.
3. Analogue filtering can be designed to fully attenuate frequency components above the nyquist bandwidth.
4. Multiple sample rates ( $F_{s1}$ ,  $F_{s2}$  and  $F_{s3}$ ) around a nominal 2.2Ms/s produce sideband components ( $F_{s1/2/3} +/- F_x$ ) well below the  $F_x/2$  frequency.
5. An algorithm unique to N4L is applied to the sideband components that derives  $F_x$  and also detects if any of the three sample rates exhibit an alias phenomenon.
6. Once  $F_x$  has been detected plus a sample rate that is known not to alias on this frequency, the PPA will track changes to  $F_x$  and knowing this frequency, a sample will always be used that will not alias.

#### Note:

- A. This technique is only needed to detect the fundamental frequency and select a non aliasing sample rate when the main frequency component of power is above 1MHz
- B. When the fundamental of power is below 1MHz, the combination of 2.2Ms/s sample rate, analogue bandwidth and asynchronous nature of noise are such that alias components will always be small or zero.
- C. The only power analyzer products presently on the market with higher sample rate than 2.2Ms/s, maintain this rate only in limited modes. Additionally, these units have a high frequency measurement error that is considerably greater than any influence on accuracy associated with potential alias components on a PPA.
- D. Products that compete with PPA series power analyzers have a sample rate to bandwidth ratio that result in a limited nyquist bandwidth, so even if our proprietary algorithms were known, they could not be applied effectively by other manufacturers.