



APPLICATION NOTE - 023

Harmonic analysis in power applications

By convention, waveforms are considered as starting ($t = 0$) at their peak values, i.e.:

$$V(t) = \cos(\omega t)$$

$$A(t) = \cos(\omega t + \theta) \quad \text{where } \theta \text{ is the relative phase angle}$$

So for harmonic analysis, using the complex notation:

$$h_n = a_n + j b_n$$

the in-phase and quadrature values of the n^{th} harmonic of a periodic waveform, $v(\phi)$, are given by:

$$a_n = \sqrt{2/2\pi} \int_{-\pi}^{\pi} v(\phi) \cos(n\phi) d\phi$$

$$b_n = \sqrt{2/2\pi} \int_{-\pi}^{\pi} v(\phi) \sin(n\phi) d\phi$$

For a square wave:

$$v(\phi) = -A \quad \text{for } -\pi < \phi < -\pi/2$$

$$= +A \quad \text{for } -\pi/2 < \phi < \pi/2$$

$$= -A \quad \text{for } \pi/2 < \phi < \pi$$

Then

$$a_n = A \sqrt{2/2\pi n} \left(\int_{-\pi}^{-\pi/2} -\sin(n\phi) d\phi + \int_{-\pi/2}^{\pi/2} \sin(n\phi) d\phi - \int_{\pi/2}^{\pi} \sin(n\phi) d\phi \right)$$

$$= A \sqrt{2/2\pi n} \left(-\sin(-n\pi/2) + \sin(n\pi/2) - \sin(-n\pi/2) + \sin(n\pi/2) \right)$$

$$= A 2\sqrt{2/\pi n} \sin(n\pi/2)$$

and

$$b_n = A \sqrt{2/2\pi n} \left(\int_{-\pi}^{-\pi/2} -\cos(n\phi) d\phi + \int_{-\pi/2}^{\pi/2} -\cos(n\phi) d\phi - \int_{\pi/2}^{\pi} -\cos(n\phi) d\phi \right)$$

$$= A \sqrt{2/2\pi n} \left(-\cos(-n\pi) + \cos(n\pi) - \cos(-n\pi) + \cos(n\pi) \right)$$

$$= 0$$

So it can be seen that for

$$n = 1, 5, 9 \dots \quad a_n = A 2\sqrt{2/\pi n} \quad (\text{relative magnitude } 1/n, \text{ phase } 0^\circ)$$

$$\text{and for } n = 3, 7, 11 \dots \quad a_n = -A 2\sqrt{2/\pi n} \quad (\text{relative magnitude } 1/n, \text{ phase } 180^\circ)$$

$$\text{and for even } n \quad a_n = 0$$

$$\text{and in all cases} \quad b_n = 0$$

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