

## APPLICATION NOTE - 023

### Harmonic analysis in power applications

By convention, waveforms are considered as starting ( $t = 0$ ) at their peak values, i.e.:

$$\begin{aligned} V(t) &= \cos(\omega t) \\ A(t) &= \cos(\omega t + \theta) \quad \text{where } \theta \text{ is the relative phase angle} \end{aligned}$$

So for harmonic analysis, using the complex notation:

$$h_n = a_n + j b_n$$

the in-phase and quadrature values of the  $n^{\text{th}}$  harmonic of a periodic waveform,  $v(\phi)$ , are given by:

$$a_n = \sqrt{2}/2\pi \int_{-\pi}^{\pi} v(\phi) \cdot \cos(n\phi) d\phi$$

$$b_n = \sqrt{2}/2\pi \int_{-\pi}^{\pi} v(\phi) \cdot \sin(n\phi) d\phi$$

For a square wave:

$$\begin{aligned} v(\phi) &= -A \quad \text{for } -\pi < \phi < -\pi/2 \\ &= +A \quad \text{for } -\pi/2 < \phi < \pi/2 \\ &= -A \quad \text{for } \pi/2 < \phi < \pi \end{aligned}$$

Then

$$\begin{aligned} a_n &= A \sqrt{2}/2\pi n \left( \underset{-\pi}{-[\sin(n\phi)]} + \underset{-\pi/2}{[\sin(n\phi)]} - \underset{\pi/2}{[\sin(n\phi)]} \right) \\ &= A \sqrt{2}/2\pi n \left( -\sin(-n\pi/2) + \sin(n\pi/2) - \sin(n\pi/2) + \sin(n\pi/2) \right) \\ &= A \sqrt{2}/\pi n \sin(n\pi/2) \end{aligned}$$

and

$$\begin{aligned} b_n &= A \sqrt{2}/2\pi n \left( \underset{-\pi}{-[-\cos(n\phi)]} + \underset{-\pi/2}{[-\cos(n\phi)]} - \underset{\pi/2}{[-\cos(n\phi)]} \right) \\ &= A \sqrt{2}/2\pi n \left( -\cos(-n\pi) + \cos(n\pi) \right) \\ &= 0 \end{aligned}$$

So it can be seen that for

$$\begin{aligned} n = 1, 5, 9 \dots & \quad a_n = A \sqrt{2}/\pi n \quad (\text{relative magnitude } 1/n, \text{ phase } 0^\circ) \\ \text{and for } n = 3, 7, 11 \dots & \quad a_n = -A \sqrt{2}/\pi n \quad (\text{relative magnitude } 1/n, \text{ phase } 180^\circ) \\ \text{and for even } n & \quad a_n = 0 \\ \text{and in all cases} & \quad b_n = 0 \end{aligned}$$