



APPLICATION NOTE - 018

Transformer Efficiency Testing and Transformer Vector Theory

Power Transformers Background

Power Transformers are used within AC power distribution systems to increase or decrease the operating voltage to achieve the optimum transmission efficiency and to provide electrical isolation between power circuits. Power is transferred via mutual induction between windings of the transformer, with the voltage being transferred from primary to secondary at the ratio of primary to secondary winding turns in accordance with the term $V_s/V_p = N_s/N_p$.

Power transformers are used to transfer power from power station to substation; the reason for this is that generation of power is very efficient at low voltages, while power transmission is more efficient at high voltages. This is because the ohmic losses (commonly known as ' I^2R ' or 'copper' losses) are significant over long distances, so power is transferred at a higher voltage with corresponding lower current.

An additional benefit of reducing the I^2R losses during transmission is that the conductor cross sectional area can be minimised. This does not come without the complications of handling high voltages, yet the advantages in efficiency outweigh the disadvantages of high voltage design.

As power levels of power transformers are now in the multiple MVA rating region, losses must be kept to an absolute minimum. Even 1% loss of a 10MVA transformer will result in a loss of 100kW

Basic Theory

If one of two adjacent windings is supplied with an alternating current source, this alternating current generates a continually changing flux surrounding the winding that induces an EMF into the second winding. With a short circuit or low impedance load connected across the second winding, current will flow in the secondary circuit.

If the two windings were only linked with air i.e. placed in close proximity to each other, the amount of flux linkage would not be enough to produce an efficient transformer and there would be a large amount of flux leakage. However, by adding a low reluctance 'core' between both windings to ensure that the maximum amount of flux is passed through the secondary, high transformer efficiency can be achieved.

So, it is from this theory we have the basic building blocks of the transformer as follows

1. An AC source
2. Primary winding
3. Low reluctance path for flux coupling (Core)
4. Secondary winding



Ideal Transformer

An ideal transformer would exhibit zero losses, i.e. no copper loss, no core loss or stray loss. The efficiency of this 'ideal' transformer would be 100% but in reality of course, this doesn't exist for reasons that we will explain in the following text. To support the theory, we will also use a high accuracy precision power analyzer to measure the losses in a real transformer. This will illustrate the importance of certain aspects of transformer loss testing instrumentation and how the Newtons4th PPA5530 offers an accurate solution to this field of testing.

Types of Losses

Copper Loss – Copper losses would be eliminated if the windings were purely inductive, however as the windings are by nature a very long piece of wire – this will have a resistive component and as voltage dropped across this resistance is gives power lost as heat. This is often referred to as the I^2R loss as Power = I^2R .

Since copper loss equates to current flow raised to the power of two, this is the dominating loss with a full load test.

Eddy Current and Hysteresis losses (also known together as 'core loss' or 'Iron Loss'). With a fixed primary voltage, core loss can be considered to be constant and therefore this is the dominating loss with a no load test.

Stray Loss – Primarily eddy current loss in nearby conductive materials induced from Leakage inductance (also referred to as a 'Flux leakage') plus skin effect losses.

Magnetostriction and Mechanical Losses – Physical movement caused by the alternating magnetic flux in a ferromagnetic material and alternating magnetic fields causing force between windings which cause heat and sound losses. There are relatively small and are usually excluded from loss measurements

Vectors

Vector analysis is sometimes steeped in mystery, however it not as complex as it first appears and offers a nice illustration of the behaviour of a power transformer. It enables the engineer to understand how each type of loss affects phase angle, impedance, and efficiency of a power transformer.

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Transformer windings are effectively Inductors

It must first be understood that a winding of a transformer acts as an inductor.



Fig.1

As seen above, V_1 is the generated voltage by the source applied to the primary winding of an ideal transformer. In the primary winding there will be a counter self EMF induced at 180 degrees to the AC source signal E_2 .

In order to generate this counter EMF the primary winding draws current, however this current is 90° lagging to the voltage source signal V_1 (We will treat the primary winding as a perfect inductor for now in this case). The below vector diagram shows I_m as the primary current (remember vectors are rotating anti-clockwise)

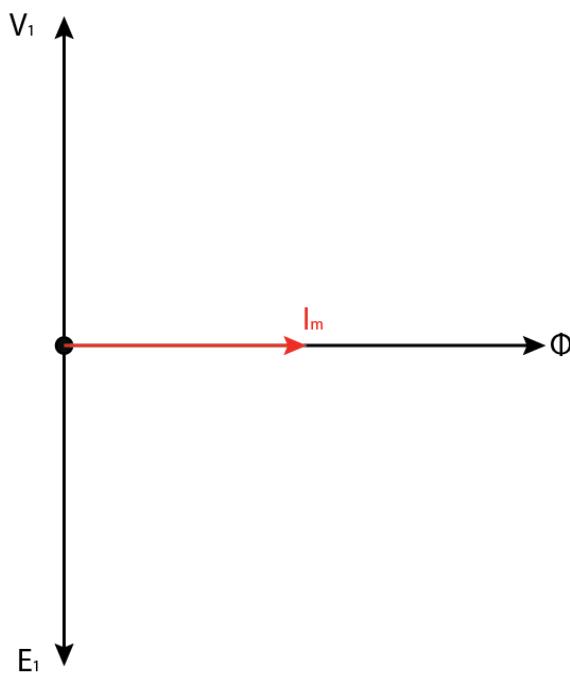


Fig.2

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This is also known as the magnetising current, i.e. it is utilised in the magnetisation of the primary coil.

The 90° phase difference is due to the following relationship

$$V(t) = L \left(\frac{dI(t)}{dt} \right)$$

Counter EMF

Counter EMF often causes a mass of confusion with electrical engineers. Counter EMF is simply a “voltage drop” across the inductor, this is the same “resistance/reactance” any load presents to a voltage source in the sense that if the EMF was not in opposition to the source it would be additive to the source voltage which we know is not the case. The difference between a resistor and an inductor is that this EMF will occur at different points in the current cycle.

As you can see the instantaneous voltage across an inductor is directly proportional to the change in current with respect to time.

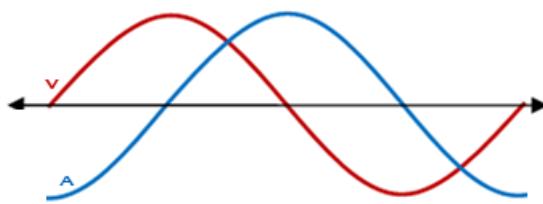


Fig.3

This can be explained as at 0 degrees in the current cycle the current is at its maximum change in value with respect to time or dI/dt (passing zero degrees). If voltage is at its maximum at this time then the voltage will be 90 degrees ahead in its waveform cycle. Therefore, the current lags the voltage by 90 degrees.

This current is utilised in order to maintain the counter EMF or voltage drop across the inductor (Or winding) and is not consumed as Watts. When the AC input signal from the grid changes polarity so does the counter EMF and the net effect is zero power, some engineers think of this reactive power as sloshing forwards and backwards with no net movement over a complete cycle.

This makes sense as with a phase angle of 90 degrees the power factor is Zero

$$\cos\theta = \cos 90 = 0$$

$$\text{watts} = V_{rms} \times I_{rms} \times PF$$

It is only when a resistive component is present that power will be dissipated within the winding; any power dissipated with an open secondary winding can be considered a loss in the core.

What is Zero Power Factor?

It is easy to state that “Zero power factor means that no real power is present, and we have a fundamental phase angle of 90 degrees, so $V_{rms} \times I_{rms} \times PF = 0$ Watts

It is also easy to dismiss what is actually happening with regards to current flow when a power factor of Zero is present. A simple explanation of the difference between reactive power and apparent power is as follows. Reactive power has Zero net effect of power transfer to a component, for every full cycle (360 degrees) the net power transfer is Zero Watts (as previously stated). This must not be misunderstood in thinking that no current is flowing.

In an inductor, during the positive half cycle the current flow is into the inductor coil. During the negative half cycle the current flow is “back into the source”, remember an inductor opposes current flow) If it were a perfect inductor (Which doesn’t exist in practice as the length of wire in the coil will exhibit a finite resistance) the net effect of power will be zero.

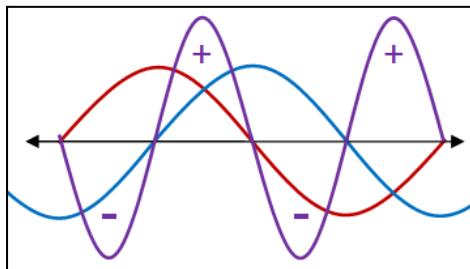


Fig.4

Flux

It is the magnetising current (which is 90° behind the source voltage) that causes the alternating flux in the core. The flux is proportional and in phase with the primary current, we can say that the primary current and magnetising flux in the core are lagging the AC source voltage on the primary by 90°. As the flux is also linked to the secondary winding – Bear in mind this is its purpose, there will also be an EMF produced by the secondary winding - this is named “mutually induced EMF”

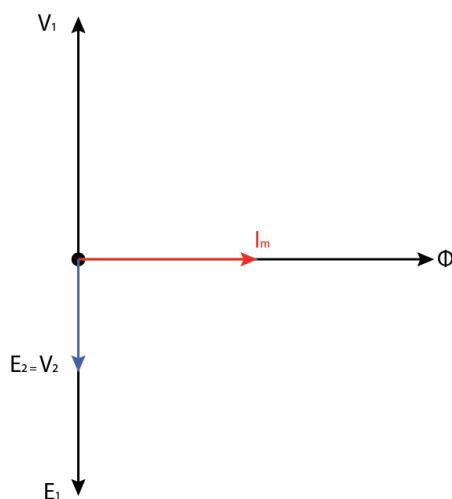


Fig.5

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As can be seen in the phasor diagram, the primary current I_m is in phase with the Alternating Flux Φ , the secondary winding is placed on the same core as the primary, this winding also generates an EMF in opposition to the alternating flux. This is in phase with the primary EMF E_1 which is 180° out of phase with the primary AC source voltage and 90° behind the alternating flux in the core. The secondary winding voltage V_2 is in phase with the secondary EMF E_2 , in this diagram the secondary is open circuit and no current will flow in the secondary.

Core Losses

Core losses are the result of Hysteresis loss and Eddy current loss, when an AC Source voltage is applied to the primary of a transformer the primary will require a current in order to magnetize the core, along with a current which will be used to overcome the losses in the core (Hysteresis and Eddy current loss)

Due to these other losses from “magnetisation of the core” The phase angle between the Primary Voltage and Primary Current is not 90° in a real world application as it would be in a perfect transformer, this is due to the core loss component effectively being an in-phase or “resistive” component with respect to the primary voltage.

As the core loss current is in phase with the primary voltage across the primary winding, it “drags” the current vector slightly closer to the voltage vector. So, we see a phase angle between the primary voltage and primary current slightly less than that of a perfect inductor of 90° .

During no load the primary current I_o in a transformer can be denoted as follows

$$I_o = I_m + i_w$$

$$I_m = I_o \sin\theta$$

$$i_w = I_o \cos\theta$$

$$I_o = \sqrt{I_m^2 + i_w^2}$$

Where I_o = Primary current, I_m = magnetizing current i_w = Core loss

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The phase diagram for this relationship is as follows

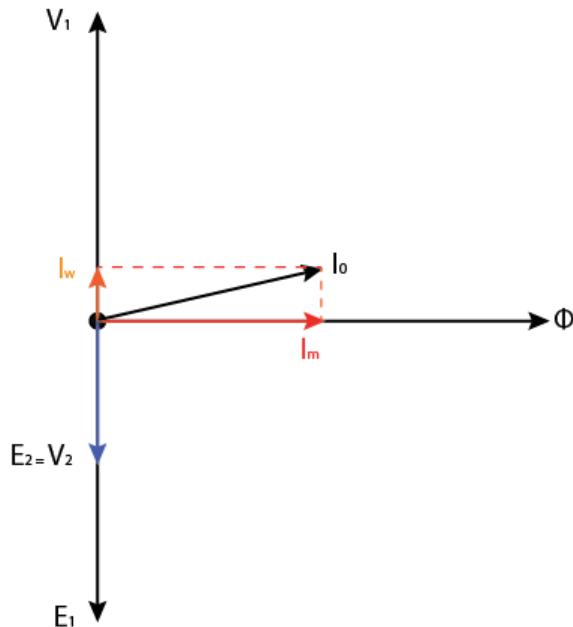


Fig.6

As you can see, I_m is typically larger than I_w and I_w is ideally as small as possible. A low value of I_w will result in a low core loss and a no-load phase angle close to 90°.

Total core loss can be described with the following equation

$$\text{Total Core Loss (Watts)} = V_1 \times I_0 \times \cos\Phi$$

Example

A 2400V/400V Transformer has a no-load current of 1A and the core loss is 500W

Determine the following

1. Power factor on the primary winding
2. Core Loss I_w
3. Magnetizing current I_m

$$V_1 = 2400V \quad V_2 = 400V \quad I_0 = 1A \quad \text{Losses} = 400W$$

$$\text{Core Loss} = 400W = V_1 \times I_0 \times \cos\Phi$$

$$400 = 2400 \times 1 \times \cos\Phi$$

$$400/2400 = \cos\Phi$$

$$0.1667 = \cos\Phi = \text{Power Factor}$$

$$\Phi = 80.41 \text{ degrees}$$

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$$\text{Magnetizing current} = I_m = I_0 \sin \Phi = 1 \sin 80.41 = 0.986 \text{ A}$$

$$\text{Core loss component} = I_w = I_0 \cos \Phi = 1 \cos 80.41 = 0.1667 \text{ A}$$

On load Phase angle

What is interesting with regards to transformer behaviour is that the phase angle of the primary, as well as the primary current magnitude is directly affected by the secondary load.

For this example, we will use a slightly inductive load on the secondary and look at the effect it has on the primary power factor. When a load is connected across the secondary winding a current I_2 flows, the resulting secondary emf acts to reduce the core flux. However, a reduction in flux also reduces E_1 , this results in an increased primary current I'_1 (balancing current) giving a restored mmf (magnetomotive force). It is important to remember that mmf remains constant on both the primary and the secondary.

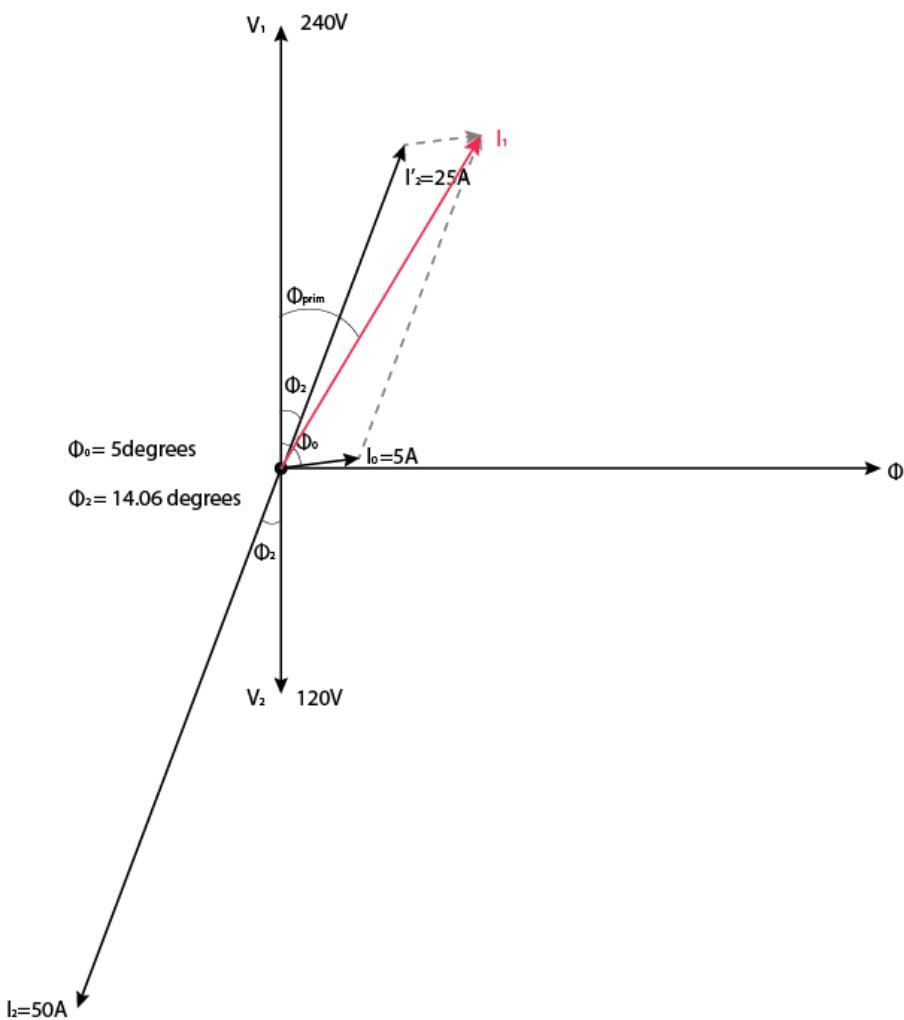


Fig.7



A 2:1 Step down Transformer is supplied with 240Vrms and has a no-load current of 5A with a power factor of 0.087 lagging. If a load with an impedance of 2.4Ω is connected to the secondary with a resultant power factor of 0.97 with respect to the secondary voltage we can then calculate the expected power factor on the primary and the primary current.

Calculate secondary voltage

$$\frac{Np}{Ns} = \frac{Vp}{Vs}$$

$$\frac{2}{1} = \frac{240}{Vs}$$

$$Vs = 120Vrms$$

Secondary Current

$$I2 = \frac{V}{R} = 50Arms$$

Secondary current Phase Angle

$$\cos^{-1} 0.97 = 14.06 \text{ degrees}$$

As this is a step-down transformer the current on the primary will be a product of the following equation

$$\frac{Ip}{Is} = \frac{Vs}{Vp}$$

$$\frac{Ip}{50} = \frac{120}{240}$$

$$Ip = \left(\frac{120}{240}\right) \times 50 = 25Arms$$

As we know the mmf is equal and opposite on the primary to the secondary which is a product of the voltage and current vectors, therefore the “balancing current” to maintain the flux on the primary required is also the same phase angle from V_1 as I_2 is from V_2

From this we know that the primary “balancing current” is 25A at a phase angle of 14.06 degrees lagging.

In order to calculate the primary power factor and total primary current we must add the balancing current and off load current I_o together vectorially.

We do this by adding the in phase and quadrature components of both vectors together as follows

$$I0 = 5A \angle 5 \text{ degrees}$$

$$\text{Vertical Component} = 5\sin 5 = 0.436$$

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$$\text{Horizontal Component} = 5\cos 5 = 4.981$$

$$I'2 = 25A \angle (90 - 14.06) \text{ degrees}$$

$$\text{Vertical Component} = 25\sin 75.94 = 24.251$$

$$\text{Horizontal Component} = 25\cos 75.94 = 6.073$$

We can add the vertical components and horizontal components to find the primary current I_1 using Pythagoras theorem

$$I_1 = \sqrt{(0.436 + 24.251)^2 + (4.981 + 6.073)^2}$$

$$I_1 = \sqrt{24.687^2 + 11.054^2}$$

$$I_1 = 27.049 \text{ Arms}$$

The resultant phase angle once the influence of the no-load current vector is accounted for can be calculated as follows

$$\tan \phi_{prim} = \frac{24.687}{11.054}$$

$$\phi_{prim} = 65.879$$

This phase angle is taken with respect to the horizontal; we must convert this to be with respect to the vertical V_1

$$\phi_{prim} = 90 - 65.879 = 24.121 \text{ degrees}$$

If we remember the secondary phase angle was 14 degrees. We can see that although the primary “balancing current” is exactly 180 degrees out of phase with the secondary current, the resultant primary current vector includes the no-load magnetizing current and core loss components.

It is not correct to assume that the impedance seen on the primary is a mirror image of the impedance connected across the secondary. The losses in the primary and magnetizing current must be known in order to subtract them from the on-load phase angle and magnitudes.



Transformer power factor and efficiency testing with the N4L PPA5530 Power Analyzer

A 1:1 isolation transformer was tested, and the results are shown in the following text, no load and max load tests were performed, short circuit tests were not carried out at this time.



Fig.8

At a mains voltage of 235.83V (fundamental 50.025Hz) and 41.796 watts the transformer has a primary off load phase angle of -77.11 degrees. This is the phase angle between V_1 and I_0

The calculated impedance at the fundamental frequency of 50.025Hz is as follows

$$235.83V / 794.49mA = 296.83\Omega$$

This is confirmed in the Impedance Analyzer mode on the N4L PPA5530.

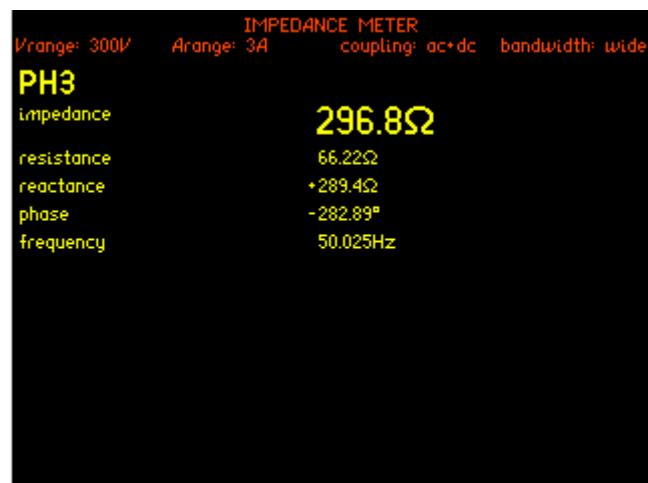


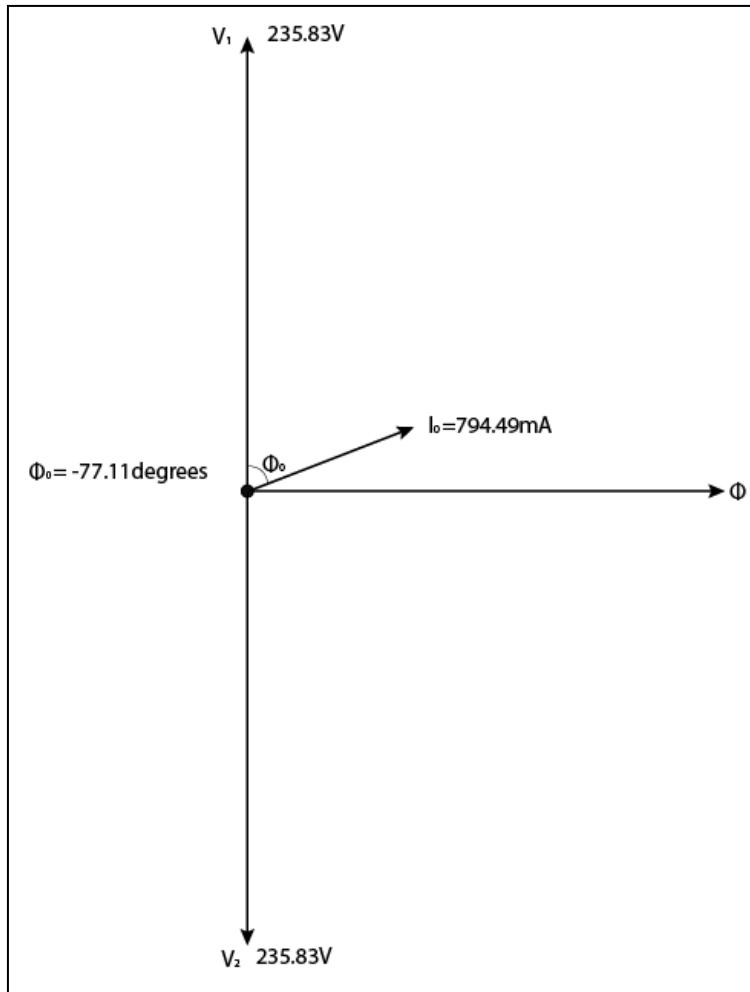
Fig.9

In order to obtain a visual representation of the PPA5530 Precision Power Analyzer's results, especially for those of us who are unfamiliar with power analyzer numerical displays which offer far more resolution than a vector display. We will convert the figures into a vector diagram to help with interpretation of the results. It must be noted that trying to read such phase angle critical



measurements from a vector diagram is the incorrect approach and the PPA5530 offers a numeric representation of the phase angles.

Note that the display will show the “fundamental impedance” This is the impedance of the transformer winding at the fundamental frequency of 50.025Hz.

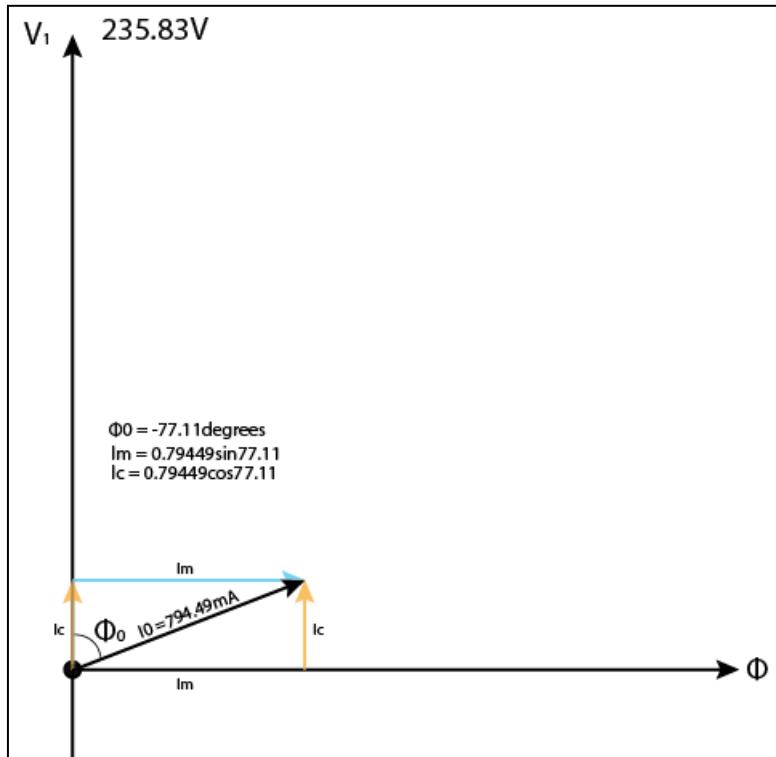


Here we can see V_1 the primary voltage from the grid, I_0 is the off-load current. This includes the magnetizing current and Core losses.

We can calculate the losses with just the primary voltage, off load current magnitude and the phase angle between the two.

We then use trigonometry to calculate the in phase current component (the losses) and multiply this by the Voltage which will give us the power in Watts. Using an N4L PPA5530 Precision Power Analyzer, all of these calculations are performed within the instrument.

This provides a good understanding of what power really is; it is the “in phase” current component providing “real” power i.e. **Heat Loss**.



As you will now be able to see, the more Hysteresis and Eddy Current loss that exists in the core, the greater the in-phase component will be.

Magnetizing Current

$$Im = 0.79449\sin 77.11$$

$$Im = 0.774A$$

Core loss component

$$Ic = 0.79449\cos 77.11$$

$$Ic = 0.177A$$

If we multiply the “In phase” core loss component with the fundamental voltage, we have the power

$$\text{Loss (Watts)} = V \times Im$$

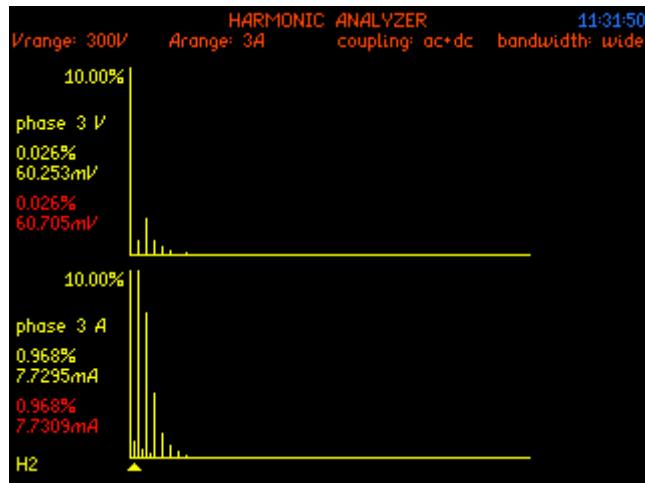
$$\text{Loss} = 235.83 \times 0.177 = 41.74W$$

Although there is a small rounding error in comparison with the correct figure shown on the PPA5530, you can see the figure does correlate with the watts measurement in figure 8.

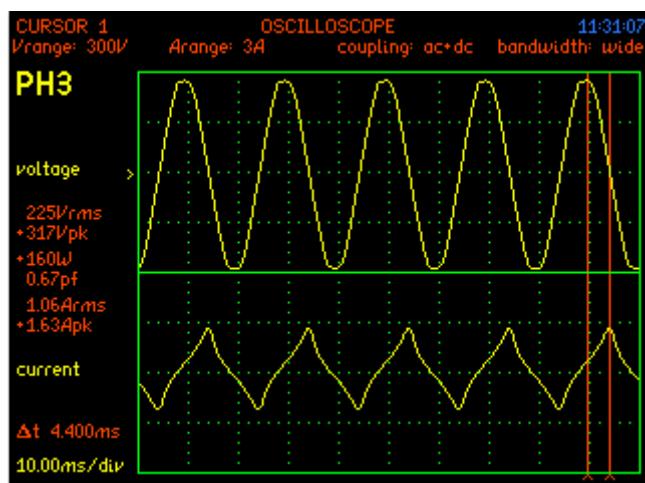


Harmonic Analysis

The N4L PPA5530 Power analyzer includes a Harmonic Analysis mode. As can be seen in the screenshot below, the voltage waveform has a small amount of distortion. (Expected of a mains waveform) interestingly the current waveform is quite heavily distorted.



This is confirmed with the Oscilloscope mode below



As seen, the current waveform is almost triangular, this likely to be a result of the manufacturer using the minimum amount of core material possible in order to save costs. We can also see an approximate Δt of 4.400ms between the current and voltage waveforms, this relates to the 11.09 degree phase angle.



On Load testing

As illustrated in figure 7 the impedance presented on the secondary winding is reflected onto the primary (taking into account core loss components). In this test a resistive heater was connected to the secondary, this load has a power factor close to unity. We will see this reflected back onto the primary and the current vector will be “pulled” closer to the mains voltage vector, this is a result of the in-phase vector presented on the secondary tending to reduce the core flux which causes an increase in the primary current which will be 180 degrees out of phase with this secondary current but in phase with the primary voltage.

On load Primary Winding - N4L PPA5530 Power Analyzer readings

Vrange: 300V		POWER	ANALYZER
Arange: 10A		coupling: ac+dc	bandwidth: wide
PH3	total	fundamental	
watts	1.4878kW	1.4868kW	
VA	1.5164kVA	1.5151kVA	
Var	-293.27Var	-291.35Var	
pf	0.9811	+0.9813	
voltage	232.61V	232.49V	+000.00°
current	6.5193A	6.5169A	-011.09°
frequency	50.012Hz		
H3	-130.23mV	-0.009%	
dc watts	-108.65μW		

Now we have 1.4868kW, 6.5169A current and a power factor of 0.9811 (-11.09° phase angle)

It must be noted that this is not quite a full load test, the transformer under test was a 1.5kVA transformer and we are testing at 1.4868kVA. In order to draw a complete vector diagram we can now test the secondary output, once the results are obtained we can now verify the theory that the impedance, the current and its respective phase angle in relation to the secondary voltage is reflected back onto the primary 180 degrees out of phase. We will also see the effect of the secondary current and its effect on the primary winding phase angle and power factor.



Secondary Winding N4L PPA5530 Power analyzer Readings

	POWER	ANALYZER	HOLD
	Vrange: 300V	Arange: 10A	coupling: ac+dc
PH2	total	fundamental	bandwidth: wide
watts	1.3840kW	1.3826kW	
V _A	1.3840kVA	1.3826kVA	
V _{Ar}	-1.7321VAr	-198.66mVAr	
pf	1.0000	+1.0000	
voltage	227.76V	227.64V	+000.00°
current	6.0766A	6.0737A	-000.01°
frequency	50.020Hz		
H3	213.51mW	0.015%	
dc watts	-445.02μW		

There is a notable difference in the two voltages on the primary and the secondary. We can see a voltage of 232.61V on the primary and 227.76V across the secondary; this is due to the voltage drop across the impedance of the secondary windings of which consists of the resistance of the windings and the leakage reactance of the windings. We have not drawn the vector components of this impedance for simplicity, but it must be taken into account.

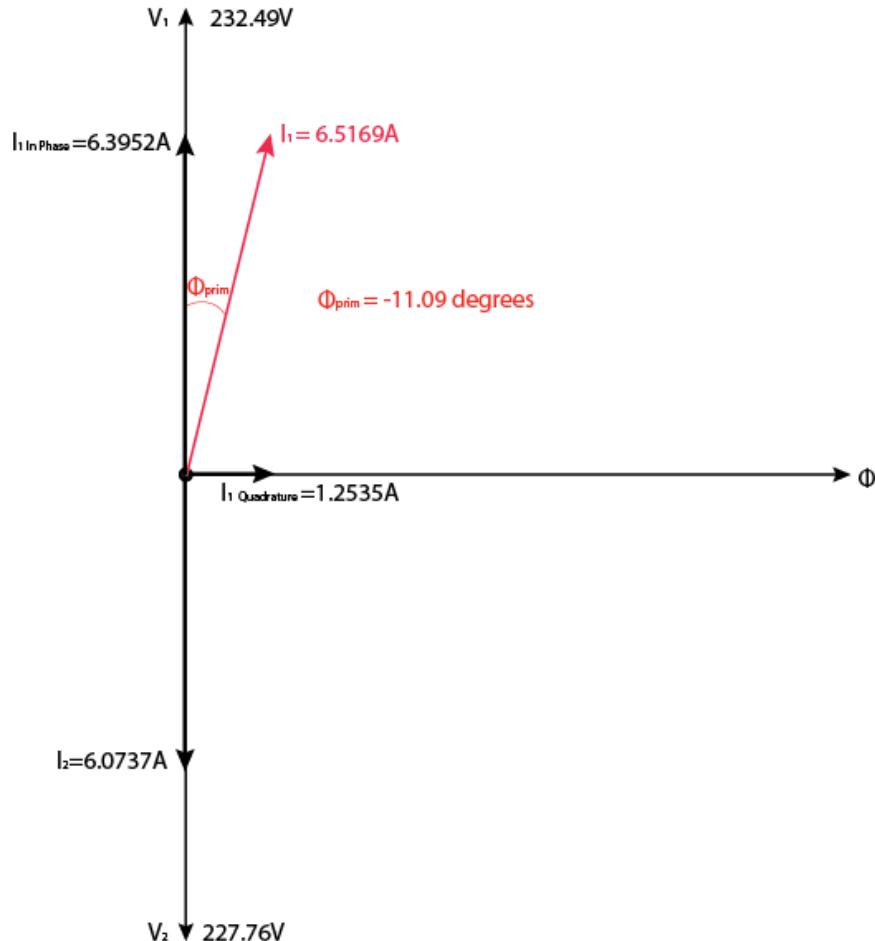
The following equations describe the relationship between the Input Voltage V_1 and the output voltage V_2

$$V1 = E1 + I1R1$$

$$V2 = E2 - I2R2$$



On load Vector diagram (simplified) for 1:1 Isolation transformer



As seen above, we can now visualise the effect that the load presented on the secondary winding will have on the input current and phase angle (or power factor) on the primary. These figures will include the core losses and copper losses discussed previously.



Efficiency Testing

Test Setup



For 3 Phase transformers two N4L PPA5530 can be used in a master slave configuration for 6 phase analysis. This test was performed on a single-phase transformer and the heat was set to maximum.

Instrument Setup

Efficiency is enabled in the *Power* menu and *phase/next phase* is selected, the input to the transformer was then connected to Phase 2 of the power analyzer and the output was connected to Phase 3, Phase 1 was not used during this test as the testing fixture was on the side of the instrument phase 3 is located.

1:1 Isolation Transformer Efficiency test results

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As seen above a result of 93% efficiency was recorded

The PPA55xx series of power analyzers provide excellent phase angle accuracy of 0.005 degrees, this offers an excellent solution for low power factor (Open secondary) core loss tests.

For more information regarding the Newtons4th (N4L) Precision Power Analyzers please visit

www.newtons4th.com