

APPLICATION NOTE - 009

Power accuracy at specified frequency and phase angle

Introduction:

The practical measurement of power is influenced by a number of factors which can introduce inaccuracies. Power analyzer manufacturers, including N4L, usually quote a simplified expression for power accuracy to allow computation of the realistic measurement uncertainty to be expected. In some applications, it may be useful to follow a more detailed process in order to more accurately quantify the worst-case theoretical measurement uncertainty under specific conditions of frequency and power factor.

Analysis:

Consider the power dissipated in a load with the following conditions:

1. sinusoidal voltage of rms magnitude V
2. sinusoidal current of rms magnitude A
3. displaced by a phase angle θ .

The power is given by:

$$W = V A \cos\theta \quad \mathbf{1}$$

Using partial derivative computation to analyse for small changes:

$$\delta W = \delta V A \cos\theta + \delta A V \cos\theta + \delta \cos\theta V A \quad \mathbf{2}$$

Dividing equation 2 by equation 1 to express this as fractions:

$$\begin{aligned} \delta W / W &= (\delta V A \cos\theta + \delta A V \cos\theta + V A \delta \cos\theta) / V A \cos\theta \\ &= \delta V / V + \delta A / A + \delta \cos\theta / \cos\theta \quad \mathbf{3} \end{aligned}$$

Consider $\delta \cos\theta$ as a result of a phase error $\delta\theta$:

$$\delta \cos\theta = \cos(\theta \pm \delta\theta) - \cos\theta$$

$$\delta \cos\theta / \cos\theta = \cos(\theta \pm \delta\theta) / \cos\theta - 1$$

so the error in the measured power is given by:

$$\delta W / W = \delta V / V + \delta A / A + (\cos(\theta \pm \delta\theta) / \cos\theta - 1)$$

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Note that as power factor tends to zero (θ tends to 90°) the uncertainty becomes dominated by the $\cos(\theta \pm \delta\theta) / \cos\theta$ term.

As these error terms are independent of each other, they may move the result in either direction:

$$\delta W / W = \pm (\delta V / V) \pm (\delta A / A) + (\cos(\theta \pm \delta\theta) / \cos\theta - 1)$$

Expressing the terms as percentage errors:

$$W_{\text{error}}\% = \pm V_{\text{error}}\% \pm A_{\text{error}}\% + (\cos(\theta \pm \delta\theta) / \cos\theta - 1) \times 100\%$$

Note that this analysis is a first order approximation assuming that the errors are relatively small.

Application:

From the measurement the following information should be noted:

1. Voltage fundamental magnitude (or rms if sinewave)
2. Current fundamental magnitude (or rms if sinewave)
3. Phase angle
4. Frequency

It may also be necessary to note the voltage and current measurement ranges.

From the manufacturer's specification compute:

1. The percentage uncertainty in the voltage measurement at the applied voltage, frequency, and voltage measurement range: $V\%$.
2. The percentage uncertainty in the current measurement at the applied current, frequency and current measurement range: $A\%$
3. The phase uncertainty at the applied frequency: $\delta\theta$

Then the percentage watts error, $W\%$, is given by

$$W\% = V\% + A\% + (\cos(\theta \pm \delta\theta) / \cos\theta - 1) \times 100\%$$

Complex waveforms:

This analysis is valid only for sinusoidal waveforms. Because the total power with a complex waveform is the result of the sum of the power of the individual harmonics, it is possible to apply this analysis to complex waveforms by

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repeating the computation for as many harmonics as desired and then combining the results.

Consider an application with a complex waveform such that only the fundamental, 3rd and 5th harmonics are significant. Then:

$$W_{\text{total}} = W_1 + W_3 + W_5 \quad \text{where: } W_{\text{total}} = \text{total power}$$
$$W_1 = \text{fundamental power}$$
$$W_3 = 3^{\text{rd}} \text{ harmonic power}$$
$$W_5 = 5^{\text{th}} \text{ harmonic power}$$

The accuracy of the power measurement for each of the harmonics can be computed individually using the procedure above. The total power measurement error then would be given by:

$$W\% = (W_1\% \times W_1 + W_3\% \times W_3 + W_5\% \times W_5) / (W_1 + W_3 + W_5)$$

Practical considerations:

The result of this analysis gives a worst-case error if all the errors of voltage, current and phase are at their worst simultaneously and in the same direction. In practice the simpler formula published in various forms by most manufacturers is a more empirical approach to quantifying power measurement uncertainty. However, the measurement uncertainty of voltage, current and phase on an instrument can be traceably quantified and the calibration certificate supplied with each instrument should be checked by the user to verify that these basic measurements are well within their specification over the whole frequency range of the instrument.

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